Mathematics II	
029	
21/11/2017	8.30 AM-11.30 AM

REB

Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2017

SUBJECT: MATHEMATICS II

COMBINATIONS:

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHEMATICS -COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHEMATICS -PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**
- **PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.
   Section A: Attempt **all** questions. **(55marks)**
   Section B: Attempt **only three** questions. **(45marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used.**
5) Use only a **blue** or **black** pen.

SECTION A : Attempt all questions. (55marks)

1)

						1						
					3		5					
				7		9		11				
			13		15		17		19			
		21		23		25		27		29		
	31		33		35		37		39		41	
43		45		47		49		51		53		55

Find the row where the number 841 would appear if the pattern above continued. **(3marks)**

2) Find the values of the constants a, b and c for which $a + b\sin 2x + c\cos 2x$ is a particular integral of the differential equation
$\frac{dy}{dx} + 4y = 20 - 20\cos 2x.$ **(4marks)**

3) A curve has a polar equation $r(4 - 3\cos\theta) = 4$. Find its Cartesian equation in the form $y^2 = f(x)$. **(4marks)**

4) Using a determinant, find the area of the triangle whose vertices are (-3, 1), (2, -4) and (5, 1). Are the given points collinear? **(3marks)**

5) In $\mathfrak{R}^3$, we have $a = (1, 0, -2)$, $b = (-1, 3, 1)$ and we consider $x = a + b$, $y = -2a + b$ and $z = 3a - 5b$. Using the properties of the vector space, calculate $T = 2x - 3y + z$. **(3marks)**

6) In a certain college, 55% of the students are female, 65% of the students are full-time and 35% of the students are male full-time. Find the probability that:
(a) a student chosen at random from all the students in the College is part-time. **(1mark)**
(b) a student chosen at random from all the students in the College is female and part-time. **(3marks)**
(c) a student chosen at random from all the female students in the College is part-time. **(2marks)**

7) Solve in $\mathfrak{R}$ set the equation: $2\ln(x+1) = \ln(1-x)$. **(3marks)**

8) If Un=$\frac{n+(-1)^n\sqrt{n}}{2n+1}$, find $\lim\limits_{n\to\infty} U_n$ $\lim_{n\to\infty} U_n$ **(4marks)**

9) From 1, 2, 3, 4, 5, 6, 7, 8, 9 copy and complete the table below such that no number (digit) is repeated and that the sum of the two digits in a column gives the third digit. **(3marks)**

10) Let f be the sine function, let g be the function $2x$ and let h be the cosine function. Prove that the function f(g) is the same as the function g(fh). **(4marks)**

11) Find the angle α between the planes with equations $2x + 3y = z - 3$ and $4x + 5y = 1 - z$. **(2marks)**

Hence write the symmetric equations of their line of intersection L. **(2marks)**

12) Find the equation of a parabola whose vertex is at the origin and directrix $x + 7 = 0$. **(3marks)**

13) a) Determine whether the series $(U_n)_{n\in IN}$ given by $U_n = \frac{2n+6}{8}$ is arithmetic or geometric. **(2marks)**

b) Calculate $\sum_{n=1}^{20} U_n$. **(2marks)**

14) The foci of hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Find the equation of the hyperbola if its eccentricity is 3. **(4marks)**

15) Calculate: $\int \frac{\tan x}{\sec x + \cos x} dx$. **(3marks)**

SECTION B: Attempt only three questions. (45marks)

16) Obtain the regression equation of 'x on y' and 'y on x' taking the origin as 2 and 200 for x and y respectively: **(15marks)**

X:	1	2	3	4	5
Y:	166	184	142	180	338

17) (a) Verify $\cot(\arctan x) = \frac{1}{x}$ for $x \neq 0$. **(4marks)**

(b) Calculate $\lim_{x\to 0}[\cos x]^{\frac{1}{x^2}}$. **(3marks)**

(c) For each complex number $u = 1 + i$ and $w = -1 + i\sqrt{3}$, find the polar form (trigonometric form). Then find z such that $z^3 = \frac{u}{w}$. **(8marks)**

18) A ladder $5\ m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate of $2\ cm/s$. How fast is its height on the wall decreasing until when the foot of the ladder is $4\ m$ away from the wall? **(15marks)**

19) The number of car accidents x in years on a highway, was found to be approximately the differential equation $\frac{dx}{dt} = kx$ where t is the time in years and k is a constant. At the beginning of 2010 the number of recorded accidents was 50. If the number of accidents increased to 60 at the beginning of 2012; estimate the number of accidents that were expected at the beginning of 2015. **(15marks)**

20) (a) Let T be a linear operator so that $T(\vec{u} + 2\vec{v}) = 8\vec{u} - 2\vec{v}$ and $T(\vec{u} - \vec{v}) = 2\vec{u} - \vec{v}$. Find $T(\vec{u})$ and $T(\vec{v})$. **(6marks)**

(b) Let f be a linear transformation so that $f(x,y) = (x + 2y, 3x - 4y)$ and $g(x,y) = (x + y, y)$. Find:

(i) $fg(x,y)$ **(3marks)**

(ii) $gof(x,y)gf(x,y)$ **(3marks)**

(iii) $f^{-1}(x,y)f^{-1}(x,y)$ **(3marks)**