

Mathematics II

029

21/11/2017

8.30 AM-11.30 AM



Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2017

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS (MCE)
- MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE (MPC)
- MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)
- PHYSICS-CHEMISTRY-MATHEMATICS (PCM)
- PHYSICS-ECONOMICS-MATHEMATICS (PEM)

DURATION: 3 HOURS

INSTRUCTIONS:

- 1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
- 2) Do not open this question paper until you are told to do so.
- 3) This paper consists of **two** sections: **A** and **B**.
 - Section A:** Attempt **all** questions. **(55marks)**
 - Section B:** Attempt **only three** questions. **(45marks)**
- 4) **Geometrical instruments and silent non-programmable calculators may be used.**
- 5) Use only a **blue** or **black** pen.

SECTION A : Attempt all questions. (55marks)

1)

						1						
					3		5					
				7		9		11				
			13		15		17		19			
		21		23		25		27		29		
	31		33		35		37		39		41	
43		45		47		49		51		53		55

Find the row where the number 841 would appear if the pattern above continued.

(3marks)

- 2) Find the values of the constants a, b and c for which $a + b \sin 2x + c \cos 2x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + 4y = 20 - 20 \cos 2x.$$

(4marks)

- 3) A curve has a polar equation $r(4 - 3 \cos \theta) = 4$. Find its Cartesian equation in the form $y^2 = f(x)$.

(4marks)

- 4) Using a determinant, find the area of the triangle whose vertices are $(-3, 1)$, $(2, -4)$ and $(5, 1)$. Are the given points collinear?

(3marks)

- 5) In \mathcal{R}^3 , we have $a = (1, 0, -2)$, $b = (-1, 3, 1)$ and we consider $x = a + b$, $y = -2a + b$ and $z = 3a - 5b$. Using the properties of the vector space, calculate $T = 2x - 3y + z$.

(3marks)

- 6) In a certain college, 55% of the students are female, 65% of the students are full-time and 35% of the students are male full-time. Find the probability that:

(a) a student chosen at random from all the students in the College is part-time.

(1mark)

(b) a student chosen at random from all the students in the College is female and part-time.

(3marks)

(c) a student chosen at random from all the female students in the College is part-time.

(2marks)

- 7) Solve in \mathcal{R} set the equation: $2 \ln(x + 1) = \ln(1 - x)$.

(3marks)

8) If $U_n = \frac{n + (-1)^n \sqrt{n}}{2n+1}$, find $\lim_{n \rightarrow \infty} U_n$ **(4marks)**

9) From 1, 2, 3, 4, 5, 6, 7, 8, 9 copy and complete the table below such that no number (digit) is repeated and that the sum of the two digits in a column gives the third digit. **(3marks)**

10) Let f be the sine function, let g be the function $2x$ and let h be the cosine function. Prove that the function $f(g)$ is the same as the function $g(fh)$. **(4marks)**

11) Find the angle α between the planes with equations $2x + 3y = z - 3$ and $4x + 5y = 1 - z$. **(2marks)**

Hence write the symmetric equations of their line of intersection L . **(2marks)**

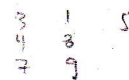
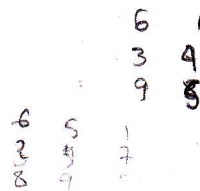
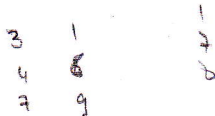
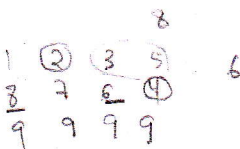
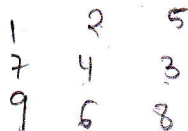
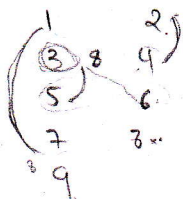
12) Find the equation of a parabola whose vertex is at the origin and directrix $x + 7 = 0$. **(3marks)**

13) a) Determine whether the series $(U_n)_{n \in \mathbb{N}}$ given by $U_n = \frac{2n+6}{8}$ is arithmetic or geometric. **(2marks)**

b) Calculate $\sum_{n=1}^{20} U_n$. **(2marks)**

14) The foci of hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Find the equation of the hyperbola if its eccentricity is 3. **(4marks)**

15) Calculate: $\int \frac{\tan x}{\sec x + \cos x} dx$. **(3marks)**



SECTION B: Attempt only three questions. (45marks)

- 16) Obtain the regression equation of 'x on y' and 'y on x' taking the origin as 2 and 200 for x and y respectively: **(15marks)**

X:	1	2	3	4	5
Y:	166	184	142	180	338

- 17) (a) Verify $\cot(\arctan x) = \frac{1}{x}$ for $x \neq 0$. **(4marks)**

- (b) Calculate $\lim_{x \rightarrow 0} [\cos x]^{\frac{1}{x^2}}$. **(3marks)**

- (c) For each complex number $u = 1 + i$ and $w = -1 + i\sqrt{3}$, find the polar form (trigonometric form). Then find z such that $z^3 = \frac{u}{w}$. **(8marks)**

- 18) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing until when the foot of the ladder is 4 m away from the wall? **(15marks)**

- 19) The number of car accidents x in years on a highway, was found to be approximately the differential equation $\frac{dx}{dt} = kx$ where t is the time in years and k is a constant. At the beginning of 2010 the number of recorded accidents was 50. If the number of accidents increased to 60 at the beginning of 2012; estimate the number of accidents that were expected at the beginning of 2015. **(15marks)**

- 20) (a) Let T be a linear operator so that $T(\vec{u} + 2\vec{v}) = 8\vec{u} - 2\vec{v}$ and $T(\vec{u} - \vec{v}) = 2\vec{u} - \vec{v}$. Find $T(\vec{u})$ and $T(\vec{v})$. **(6marks)**

- (b) Let f be a linear transformation so that $f(x, y) = (x + 2y, 3x - 4y)$ and $g(x, y) = (x + y, y)$. Find:

(i) $fg(x, y)$ **(3marks)**

(ii) $gof(x, y)$ **(3marks)**

(iii) $f^{-1}(x, y)f^{-1}(x, y)$ **(3marks)**